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TECHNICAL NOTES

Effect of aspect ratio on mass transfer from a rotating cup

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1. INTRODUCTION

Reference [1] reports on a numerical study of mass transfer characteristics for a cup-like cylindrical vessel, which rotates about its central longitudinal axis. Attention was focused on the enhancement of transfer properties, caused by the rotation, from the inner surface walls of the cup to the surrounding fluid. The numerical solutions of [1] to the governing Navier–Stokes equations provided detailed data on the local Sherwood number Sh as well as the average Sherwood number $\overline{Sh}$. The earlier experiment of [2], by using a naphthalene sublimation technique, gave an empirically-constructed formula for $\overline{Sh}$ as functions of Re and (L/R) in the range up to $Re \gtrsim O(10^3)$. In the above, Re denotes the rotational Reynolds number of the cup [$\equiv \omega R^2/\nu$], where ω is the rotation rate of the cup, R the inner radius, and ν the kinematic viscosity of the fluid, L the height of the cup. It was shown that the numerical results of [1] for $\overline{Sh}$ were in close agreement with the experimental findings of [2].

In this note, the methodologies of [1] are extended to explore the explicit impact of the aspect ratio (L/R) on the transfer properties. In [2], only values of (L/R) of order unity were considered. It is stressed that the transport characteristics are heavily influenced by the geometrical constraints, and the aspect ratio is a primal factor in determining the container shape. The purpose of this note is to examine the variations of Sh and $\overline{Sh}$ when (L/R) encompasses a wide range. In particular, extreme values of (L/R) are of special concern. For these cases, the global flow patterns as well as the attendant transfer properties show qualitatively different features. In light of the present numerical results, the validity of the empirical correlation formula of [2], which is based on measurements for (L/R) of O(1), will be assessed in a broader parameter space of Re and (L/R).

2. RESULTS AND DISCUSSION

The flow configuration, numerical solution procedures, and the treatment of boundary conditions have been fully explained in [1], and they will not be reproduced here. In the present computations, a total of six cases of $L/R = 0.25$, 0.5, 1.0, 2.0, 4.0, 6.0, were adopted.

The flow and concentration fields surrounding the cup walls at high Re are exemplified in Figs. 1 and 2. In a similar manner to ref. [1], the angular velocity ($\equiv v/r$) is shown in frame (a), and the meridional stream function ψ in frame (b). Here, ψ is defined such that $u = (1/r)\,(\partial\psi/\partial z)$ and $w = -(1/r)\,(\partial\psi/\partial r)$. The concentration field C is displayed in frames (c) and (d) for type A [non-transferring endwall base] and type B [constant-concentration and endwall base], respectively. Note that concentration is constant [$C = 1.0$] at the cylindrical wall surface.

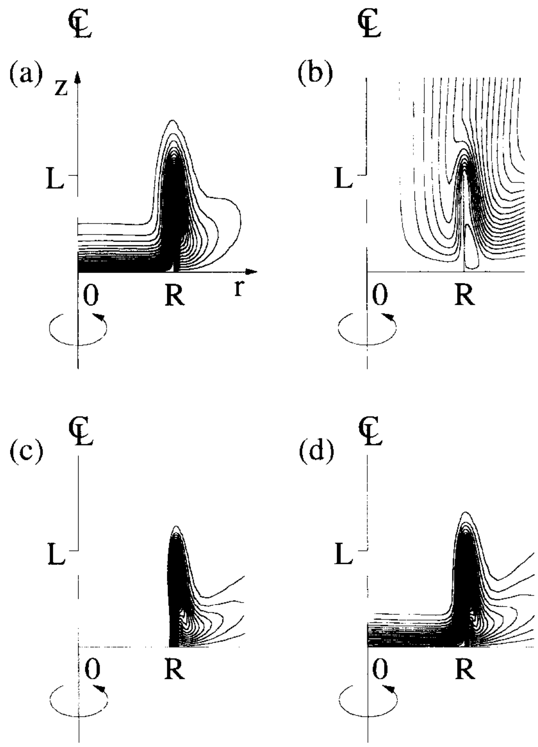

Fig. 1. Depiction of flow field. $Re = 800$, $L/R = 0.25$. The number of contours is 20. (a) Angular velocity (v/r) field. (b) Meridional stream function ψ, nondimensionalized by $R^3\omega$. (c) Concentration (C) field for type A. (d) Concentration (C) field for type B.

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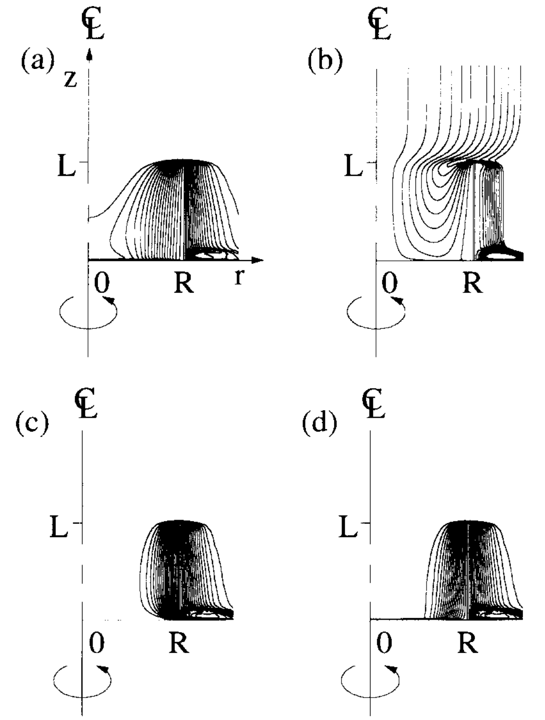

Fig. 2. As Fig. 1, except for $L/R = 6.0$.

Figure 1 [for $L/R = 0.25$] is typical of the results for small aspect ratios. The rotating cup is shallow in depth, and the angular velocity in the bulk of the cup interior is uniform in the radial direction. In the axial direction, the angular velocity is conspicuous only in a narrow boundary layer adjacent to the endwall base, and v/r diminishes rapidly as z increases away from the boundary layer. At large radii (v/r) increases to meet the no-slip condition of the cylindrical sidewall. The behavior of (v/r) and ψ in much of the central radial locations is akin to that of a rotating flat disk. The fluid is drawn into the cup interior, due to the action of the rotating endwall base, in the central portion; in order to satisfy the mass continuity, the fluid moves toward the opening of the cup in the region close to the cylindrical sidewall. The concentration field for type A indicates that convective activities are minimal in the bulk of the interior, and strong radial gradients of concentration are seen in the close vicinity of the sidewall. For a transferring base [type B], the overall transport process is heavily influenced by conduction, since the global convective transfers are weak.

The flow structure for moderate L/R was extensively discussed in [1], and the reader is referred to that paper for details. Figure 2 is illustrative of the case of a large L/R. Clearly, the radial dependence of the angular velocity is discernible. The depth of the cup(L) is much larger than the radius of the cup(R); the interior fluid flow is determined primarily by the cylindrical sidewall. In the bulk of the interior, v/r increases radially. In an extremely thin layer adjacent to the endwall base, v/r undergoes very steep changes to accommodate the no-slip condition at the solid wall. The meridional flow displays substantial variations in the radial as well as in the axial directions. As demonstrated in frames (c) and (d), the difference in C-field between type A and type B is relatively small. This is not unexpected in view of the fact that, for a deep cup, the impact of the base on the global flow characteristics is minor. The C-field is determined primarily by the conditions of the cylindrical sidewall. Only in the region very close to the base, the C-field displays appreciable difference between type A and type B. In the bulk of the central interior region, the meridional flow is not vigorous; therefore, the iso-C lines tend to be axially parallel, which implies that the flow is under heavy influence of conductive transport from the cylindrical sidewall. In passing, it is worth mentioning briefly the flow pattern in the exterior region of the cup. The stated objective of this paper is to study the forced convection from the interior surface of the cup; therefore, as remarked earlier, the solutions for the exterior regions are not of direct interest. However, the computed flow patterns in the bottom portions of the exterior regions are suggestive of the Taylor–Couette vortex (see e.g. ref. [3]). This flow pattern is more pronounced when L/R becomes large, as in Fig. 2. The mass transfer from the external surface of the cup is not a subject of the present account; it will be dealt with in subsequent endeavors.

Based on the computed flow data, the variance of local Sherwood number Sh is examined in Fig. 3. First, for type A (non-transferring base), Sh_c at the cylindrical sidewall is shown. The mass transfer coefficient Sh_c over much of the cup height diminishes appreciably as the cylinder aspect ratio increases beyond O(1). Similar behavior is noted for Sh_c for type B (transferring base), although the rate of decrease of Sh_c with L/R is less pronounced for type B than type A. The Sherwood number Sh_b at the endwall base in the central interior portion demonstrates minor dependence on L/R. The role of the base endwall disk weakens for large L/R, which results in a reduction of Sh_b at large radii.

Finally, summarizing the entire numerical data, the vari-

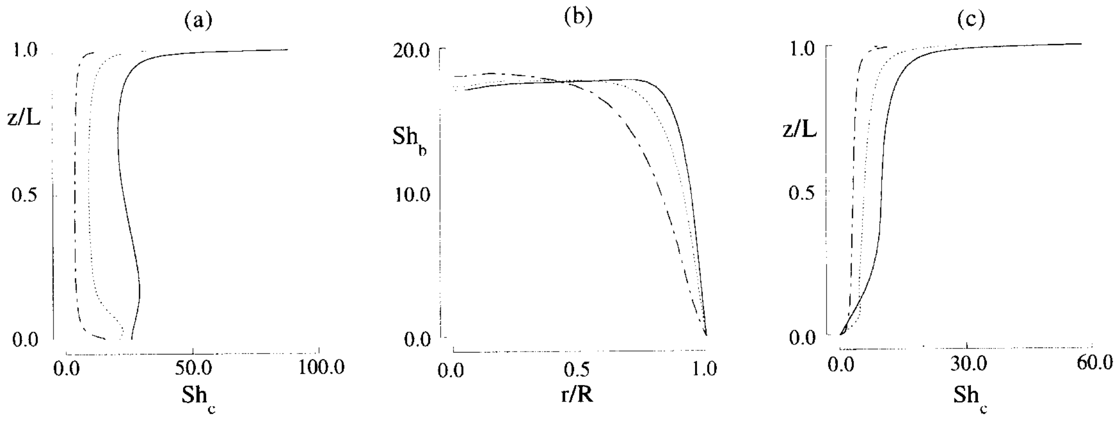

Fig. 3. Distributions of the local Sherwood number. $Re = 800$. (a) Local Sherwood number Sh_c at the cylindrical sidewall. Type A. (b) Local Sherwood number Sh_b at the base disk. Type B. (c) Local Sherwood number Sh_c at the cylindrical sidewall. Type B. —, $L/R = 0.25$; $\cdots$, $L/R = 1.0$; — · —, $L/R = 6.0$.

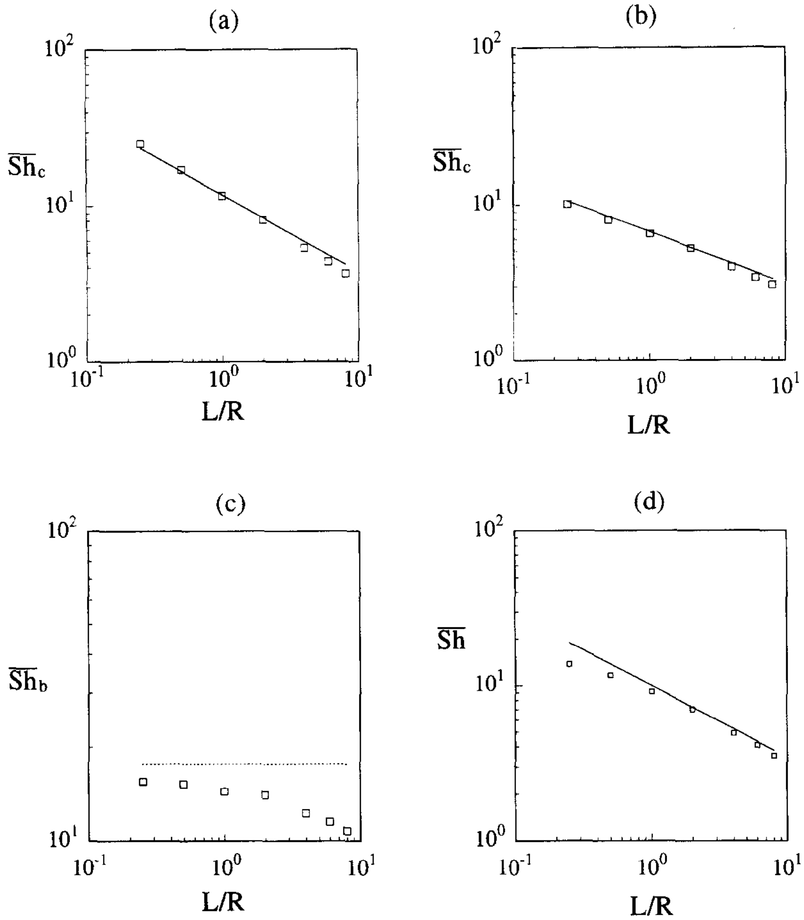

Fig. 4. Averaged Sherwood number $\overline{Sh}$ vs L/R. $Re = 800$. (a) $\overline{Sh}_c$ at the cylindrical wall. Type A. (b) $\overline{Sh}_c$ at the cylindrical wall. Type B. (c) $\overline{Sh}_b$ at the base disk. Type B. (d) $\overline{Sh}$ at the entire wall surface. Type B. —, experimental results of ref. [2]; □, present numerical data; ···, result for a rotating disk of ref. [4].

ation of the average Sh with L/R is scrutinized in Fig. 4. Comparisons are made between the present numerical results and the empirically-fitted formula of [2]. Cross-checks of the results establish that the empirical formula tends to portray the average Sherwood number with reasonable accuracy. This comparison gives credence to the applicability of the empirical relation over a wider range of L/R than that actually used in the experiment. In Fig. 4(c), for comparison purposes, the numerical data for $\overline{Sh}_b$ are checked against the results of an isolated flat disk rotating in its own plane (ref. [4]). Evidently, $\overline{Sh}_b$ for a cup, as L/R diminishes, approaches that for a flat disk.

3. CONCLUSION

When L/R is very small, the flow patterns are qualitatively similar to those for a rotating flat disk. Convective activities are meager in the bulk of the interior for a non-transferring base endwall disk. For a transferring base, conduction plays a significant role.

When L/R is large, the interior flow is determined chiefly by the cylindrical sidewall. Only in an extremely thin boundary layer on the base disk, velocities undergo rapid changes. Differences in the concentration fields are generally small between the two types of the base conditions.

The local mass transfer coefficient at the cylindrical sidewall, Sh_c, becomes very small over much of the cup height when $L/R \gtrsim O(1)$. However, this tendency is less pronounced for type B than for type A. Also, as L/R increases, the influence of the base disk weakens, which results in a reduction in Sh_b at the base.

Cross-comparisons with the numerical data establish that the empirically-obtained formula for $\overline{Sh}$ is applicable in a broader range of L/R.

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